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Reheating of the Universe as holographic thermalization

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ABSTRACT

Assuming gauge/gravity correspondence we study reheating of the Universe using its holographic dual. Inflaton decay and thermalisation of the decay products correspond to collapse of a spherical shell and formation of a blackhole in the dual anti-de Sitter (AdS) spacetime. The reheating temperature is computed as the Hawking temperature of the developed blackhole probed by a dynamical boundary, and is determined by the inflaton energy density and the AdS radius, with corrections from the dynamics of the shell collapse. For given initial energy density of the inflaton field the holographic model typically gives lower reheating temperature than the instant reheating scenario, while it is shown to be safely within phenomenological bounds.

According to the standard lore of inflationary cosmology, reheating of the Universe is caused by out-of-equilibrium decay of the inflaton field that oscillates about its potential minimum. Although this is a crucial process that determines the subsequent thermal history of the Universe, our understanding of it is still incomplete as the decay process down to the Standard Model (SM) particles is highly involved. There are several phenomenological models of reheating, providing different approaches to evaluate the reheating temperature. Among these, the most traditional one is based on perturbative Born decay of the inflaton and the reheating temperature is computed from the condition that the inflaton decay rate Γ becomes comparable to the Hubble expansion rate H, as

$$T_{\rm pert} \approx \left(\frac{90}{\pi^2 g_*}\right)^{\frac{1}{4}} (M_{\rm P}\Gamma)^{\frac{1}{2}} . \tag{1}$$

Here, g_* is the relativistic degrees of freedom at the time of reheating, $M_{\rm P} \equiv (8\pi G_4)^{-1/2} = 2.4 \times 10^{18}$ GeV is the reduced Planck mass and G_4 is the four-dimensional Newton constant. This Born decay picture is known to be too simplistic, at least in some cases, as nonperturbative resonance effects can change the decay rate drastically. In the scenario of *preheating* [1], reheating is assumed to take place in three steps: the nonperturbative resonant decay of the inflaton, followed by perturbative cascade decay of the decay products, and then eventual thermalisation. There exist proposals of other reheating mechanisms, including those based on evaporation of primordial blackholes [2], surface evaporation of Q-balls [3], and nonminimal gravitational coupling of the inflaton [4]. We discuss, in this Letter, a novel description of reheating based on gauge/gravity correspondence [5,6]. This may be considered as the limit opposite to the perturbative scenario and is supposed to take account of strongly coupled dynamics in the thermalisation process.

Following the idea of holographic thermalisation [6,7] which asserts that blackhole formation in a $(d+1)$-dimensional anti-de Sitter (AdS) spacetime is a dual description of out-of-equilibrium thermalisation in d-dimensional conformal field theory (CFT), we postulate that the Universe sits at the boundary of a five-dimensional asymptotically AdS spacetime. We shall consider, schematically, the boundary action of the form

$$S_{\rm bdry} = S_{\rm CFT} + \int d^4x \Phi_0(\tau)\mathcal{O}(\tau), \tag{2}$$

and regard $S_{\rm CFT}$ as the action of the Universe including (but not restricted to) the SM matter. Here we treat the inflaton as an external field that is not included in the matter of the Universe. The operator $\Phi_0(\tau)$ represents the oscillating inflaton and $\mathcal{O}(\tau)$ is the matter in the Universe that couples to the inflaton.[1] Aside from

[1] In the two-body scattering into two bosons $\phi\phi \to \chi\chi$, for example, $\Phi_0 = \phi^2$ and $\mathcal{O} = \chi^2$. In the case of Higgs inflation the Higgs field ought to be split into the massive (inflaton) part Φ_0 and the nearly massless (SM) part which is in the CFT.

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the interaction with the inflaton, the matter content of the Universe is nearly massless at high energies and may be modelled as a CFT. Prior to reheating the Universe must have undergone a rapid adiabatic expansion, i.e. inflation. Therefore the CFT is at zero-temperature when reheating commences. Our use of holography is motivated by the success of holographic quantum chromodynamics (QCD) [8]; the energy scale of reheating may well be higher than that of the quark-hadron phase transition, and then the "radiation" in the Universe should consist of ultra-relativistic quark-gluon plasma. We will not, nevertheless, specify the particle content of the CFT below. Although a legitimate use of gravity dual would require e.g. a large number of colours N, we will take a phenomenological approach and assume the existence of the gravity dual. Our focus here is on what the gravity dual will tell us about reheating of the Universe.

The out-of-equilibrium decay of the inflaton is a process of transferring its energy to the matter in the Universe. This may be seen as disturbance of the CFT by external shock represented by the oscillating inflaton operator $\Phi_0(\tau)$ in (2). The time scale of the disturbance $\Delta\tau$ may be determined by the decay efficiency and Hubble damping. In the gravity dual, the thermalisation corresponds to formation of a blackhole in AdS$_5$, caused by collapse of a shell that destabilises the pure AdS. The thickness of the shell corresponds to the time scale of reheating $\Delta\tau$. The boundary conditions of the infalling shell should be given by the oscillating field $\Phi_0(\tau)$ of the boundary action (2), in accordance with the GKPW prescription [5,6].

The dynamics of blackhole formation in the asymptotically AdS spacetime is described by the AdS–Vaidya solution [9],

$$ds^2 = -f(r,v)dv^2 + 2dvdr + r^2 d\Omega_3^2,$$
$$f(r,v) = 1 + \frac{r^2}{L^2} - \frac{r_0^2}{r^2}\theta(v), \tag{3}$$

where L is the AdS radius and r_0 is related to the mass of the five-dimensional blackhole by

$$M_5 = \frac{3\pi r_0^2}{8G_5}. \tag{4}$$

Here, G_5 is the five-dimensional Newton constant. The function θ asymptotes to $\theta \to 0$ inside the shell and $\theta \to 1$ outside, and thus the AdS–Vaidya solution interpolates the pure AdS solution in the past (inside the shell) and the AdS–Schwarzschild solution in the future (outside). With the change of variables $dv = dt + f(r,v)^{-1}dr$, the metric in the static coordinates reads

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_3^2, \tag{5}$$

in which the function $f(r)$ behaves as

$$f(r) \to \begin{cases} f_-(r) \equiv 1 + \frac{r^2}{L^2} & \text{(inside)}, \\ f_+(r) \equiv 1 + \frac{r^2}{L^2} - \frac{r_0^2}{r^2} & \text{(outside)}. \end{cases} \tag{6}$$

After the shock passes, the metric seen by a local observer becomes AdS–Schwarzschild, indicating that the CFT at the boundary is thermalised. The temperature of the CFT will be given by the Hawking temperature of the AdS–Schwarzschild blackhole, which may be interpreted as the reheating temperature of the Universe.

Cosmological application of holography has been actively studied since the early days of AdS/CFT correspondence. If we are to consider the Friedman–Robertson–Walker (FRW) universe as the CFT side of the correspondence, we are faced with two apparent obstacles. An expanding universe is weakly gravitating and hence the boundary theory in such a setup is not entirely decoupled

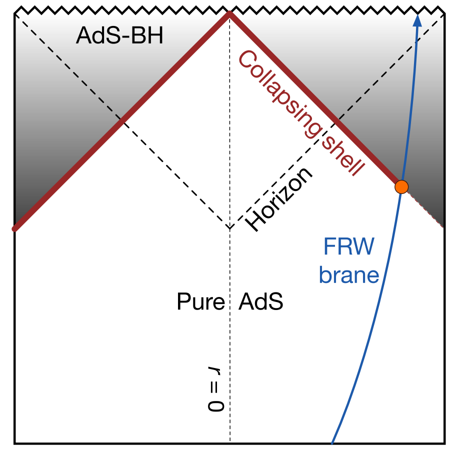

Fig. 1. The Penrose diagram of the AdS–Vaidya solution describing blackhole formation. The Universe is considered as a probe FRW brane, a hypersurface solving Israel's junction conditions. The collapsing shell is released form the brane during reheating, with boundary conditions given by $\Phi_0(\tau)$. The region outside the FRW brane is to be excised so that the brane represents a true boundary of the spacetime. (For interpretation of the colours in this figure, the reader is referred to the web version of this article.)

from gravity. The other issue is the time dependence of the temperature; in contrast to the standard flat space CFT case in which the overall scaling of the temperature is unfixed, in cosmology the temperature has a definite value and redshifts as the inverse of the scale factor, $T \propto a^{-1}$. These features suggest that when discussing cosmology in AdS/CFT, the boundary theory should be treated dynamically [10]. The Universe is then envisaged as a hypersurface moving in the asymptotically AdS$_5$ bulk.

To proceed, we make use of the observation [11,12] that the Friedman equation is obtained from the induced metric on a hypersurface in the five-dimensional AdS–Schwarzschild (or AdS–Vaidya) spacetime. The emerging Friedman equation is

$$H^2 = -\frac{1}{a^2} + \frac{r_0^2}{a^4} + \cdots, \tag{7}$$

where the ellipses represent terms that come from extra matter on the brane, which are not important in our discussion of reheating and will be neglected. The second term in (7) is a radiation-like contribution proportional to the mass of the five-dimensional blackhole. In many of the brane universe literature this term is treated as an extra contribution *in addition to* the matter of the Universe, but here in holographic reheating this term is naturally interpreted as the thermal radiation resulting from thermalisation of the shock. The first term of (7) is the curvature term $-k/a^2$, indicating that we are considering the closed ($k = 1$) FRW universe.

Identification of the FRW metric and the induced metric on the hypersurface implies that the scale factor of the universe coincides with the AdS radial coordinate, $a = r$. An expanding universe is thus a brane moving away from the centre of the AdS. Fig. 1 shows embedding of the FRW universe in the AdS–Vaidya spacetime. Reheating takes place at the transition from the pure AdS to the AdS–Schwarzschild background, marked by the small orange circle. As we regard the FRW brane as a true boundary of the spacetime, the region to the right of the brane is understood to be cut away. The renormalisation scale is thus time-dependent in the global coordinates. In the unit of the boundary (FRW) time, however, the renormalisation scale is approximately constant as the warp factor is trivial. The collapsing shell is released from the brane with the boundary conditions given by $\Phi_0(\tau)$. The horizon develops as the shell collapses. Its location $r = r_+$ is found as a solution to $f_+(r) = 0$,

$$r_+ \equiv L\left[\frac{1}{2}\left(\sqrt{1+\frac{4r_0^2}{L^2}}-1\right)\right]^{\frac{1}{2}}. \tag{8}$$

The Hawking temperature of the blackhole seen by a static observer is computed in the standard way by Euclideanising the near-horizon metric. The absence of a conical singularity then gives

$$T_{\text{static}}(r) = \frac{2r_+^2+L^2}{2\pi L^2 r_+}\frac{1}{\sqrt{f_+(r)}}, \tag{9}$$

where the factor $1/\sqrt{f_+(r)}$ is due to gravitational redshift (the Ehrenfest–Tolman effect). This T_{static} cannot be the temperature of the probe brane as it is ill-behaved near the horizon. Nevertheless, it should coincide with the temperature of the probe brane when the probe brane is far outside the horizon and nearly static. Thermodynamics on the brane suggests that the natural time scale on the moving brane is $d\tau = \frac{a}{L}dt$ [12], from which the temperature of the probe brane is found as

$$T_{\text{probe}} = \frac{2r_+^2+L^2}{2\pi L r_+}\frac{1}{a}. \tag{10}$$

This is regular at the horizon and coincides with (9) when $a \gg r_0$, $a \gg L$. Hence (10) is qualified to be the temperature of the FRW universe.

Apart from the scale-factor dependent redshift, T_{probe} is determined through (4) and (8) by the five-dimensional blackhole mass M_5 that encodes the physics of inflaton decay. While details of the reheating process may be involved, in the gravity dual energy conservation is expected. As the blackhole results from the collapse of the shell, it is natural to assume

$$M_5 = \varepsilon \times (\text{area of shell})_* \times (\text{energy density of shell})_*$$

where ε is the efficiency ($0 < \varepsilon \le 1$) of blackhole formation and the asterisk denotes quantities evaluated at the onset of reheating. The shell is spherical and its area is $2\pi^2 r_*^3 = 2\pi^2 a_*^3$. The energy density of the shell is related to that of the oscillating inflaton ρ_* at the onset of reheating. As we consider the closed universe, the total inflaton energy is finite and is given by $2\pi^2 a_*^3\rho_*$. Taking into account the redshift a_*/L of the energy between the brane and the shell frame,[2] the blackhole mass is written as

$$M_5 = \frac{2\pi^2\varepsilon a_*^4\rho_*}{L}. \tag{11}$$

Given a model of inflation, the inflaton energy density ρ_* may be evaluated explicitly. For an inflaton field φ with mass m and negligible self-interaction, for example,

$$\rho_* = 3M_P^2H_*^2 = \left[\frac{1}{2}(\partial_\tau\varphi)^2 + \frac{1}{2}m^2\varphi^2\right]_* \approx m^2\varphi_*^2, \tag{12}$$

with φ_* the initial amplitude of the oscillating inflaton.

To interpret (10) as the temperature of the Universe, it is important to note that the phase structure of the thermodynamics of an AdS–Schwarzschild blackhole is entirely different from the asymptotically flat Schwarzschild case. When $r_0 \ll L$, we find $T_{\text{probe}} \approx L/2\pi r_0 a$. This phase is similar to the asymptotically flat case and exhibits instability due to negative specific heat; clearly, it does not represent the thermal equilibrium of the Universe. Taking the opposite limit $r_0 \gg L$, which is equivalent to choosing large a_* and is natural after inflation, the temperature (10) behaves as

$$T_{\text{probe}} \approx \frac{r_0^{\frac{1}{2}}}{\pi L^{\frac{1}{2}} a}. \tag{13}$$

The specific heat in this phase is positive, appropriate for the reheating model. This is the deconfinement phase of QCD. Now using (4) and (11) in (13) the temperature of the FRW universe is expressed as

$$T = \left(\frac{8G_5M_5}{3\pi^5L^2}\right)^{\frac{1}{4}}\frac{1}{a} = \left(\frac{16G_5\varepsilon\rho_*}{3\pi^3L^3}\right)^{\frac{1}{4}}\frac{a_*}{a}. \tag{14}$$

Denoting the scale factor at the moment of thermalisation (end of reheating) as a_{rh}, the reheating temperature is written as

$$T_{\text{holo}} = \left(\frac{16G_5\varepsilon\rho_*}{3\pi^3L^3}\right)^{\frac{1}{4}}\frac{a_*}{a_{\text{rh}}}. \tag{15}$$

This may be rewritten using the relation $G_5 = G_4L/2 = L/16\pi M_P^2$ [10] between the four- and five-dimensional Newton constants as

$$T_{\text{holo}} = \left(\frac{\varepsilon\rho_*}{3M_P^2L^2}\right)^{\frac{1}{4}}\frac{a_*}{\pi a_{\text{rh}}}. \tag{16}$$

Thus in the holographic model, the reheating temperature is determined by the inflaton energy density ρ_*, the AdS radius (characterising the CFT) L, as well as by the efficiency of the blackhole formation ε and the redshift during reheating a_*/a_{rh}. The efficiency ε depends on details of the collapsing dynamics and may be evaluated numerically. The redshift a_*/a_{rh} is related to the function $\theta(v)$ of (3) that gives the thickness of the shell, as a_* (a_{rh}) is the value of the scale factor when the leading (trailing) edge of the shell is released from the brane. As most of the shell energy is expected to be used in blackhole formation and the Hubble expansion during reheating is not large, it is natural to suppose that both ε and a_*/a_{rh} are not much smaller than $\mathcal{O}(1)$.

To illustrate our results, let us compare this scenario with instant reheating in which the inflaton energy density ρ_* is immediately converted into the energy density of radiation. The temperature of instant reheating follows from the Stefan–Boltzmann law and reads

$$T_{\text{inst}} = \left(\frac{30\rho_*}{\pi^2 g_*}\right)^{\frac{1}{4}}. \tag{17}$$

Comparing (15) and (17), we may define the effective degrees of freedom for holographic reheating,

$$g_*^{\text{eff}} = \frac{45\pi L^3}{8G_5\varepsilon}\left(\frac{a_{\text{rh}}}{a_*}\right)^4. \tag{18}$$

The L^3 dependence is natural since the central charge of strongly coupled four-dimensional CFT in deconfinement phase is $c \sim N^2 \sim L^3/G_5$.

As an example, let us consider the $m^2\varphi^2$ chaotic inflation model with the Planck-normalised inflaton mass $m = 1.5\times 10^{13}$ GeV. The amplitude of the oscillating inflaton is $\varphi_* \approx \sqrt{2}M_P$ at the end of slow roll and the inflaton energy density (12) is $\rho_* \approx 8\times 10^{-11}M_P^4$. Using the relativistic degrees of freedom $g_*^{\text{SM}} \sim 100$ of the SM, the instant reheating scenario yields somewhat high reheating temperature $T_{\text{inst}} \sim 3\times 10^{15}$ GeV. In the holographic scenario, the effective degrees of freedom (18) is written using (16) as

$$g_*^{\text{eff}} = \frac{90\pi^2M_P^2L^2}{\varepsilon}\left(\frac{a_{\text{rh}}}{a_*}\right)^4. \tag{19}$$

Since $\varepsilon \le 1$ and $a_{\text{rh}} > a_*$, and as the AdS radius must be larger than the Planck length $M_P L \gtrsim 1$, the holographic effective degrees of

[2] The cosmic time on the brane before and after the transition may differ. However, the difference is immaterial as our interest is only in the asymptotic region where $f_-(a) \approx a^2/L^2 \approx f_+(a)$.

freedom $g_*^{\rm eff}$ is larger than $g_*^{\rm SM}$. Correspondingly, the holographic reheating temperature $T_{\rm holo}$ is lower than $T_{\rm inst}$, given the same energy density of the decaying inflaton. How large can $g_*^{\rm eff}$ be? The nucleosynthesis bound of the reheating temperature is $T_{\rm rh} \gtrsim$ a few MeV [13]. However, for the holographic model it is more appropriate to take the quark-hadron phase transition ~ 200 MeV as the lower bound of the strongly coupled CFT temperature; this gives $g_*^{\rm eff} \lesssim 5 \times 10^{66}$. Thus, sufficiently large parameter space is left for the scenario of holographic reheating. To discuss constraints on the parameters by observation, the Bayesian analysis of [14] may be useful.

In QCD, $N = 3$, $c \sim 10$ and the effective degrees of freedom is $g_*^{\rm eff} \sim 100\,\varepsilon^{-1}(a_{\rm rh}/a_*)^4$. The AdS radius in this case is close to the Planck scale, which is a common feature of holographic QCD. Our description by confining theory is natural when the reheating temperature is about the QCD scale; this corresponds to $\rho_*^{1/4} \sim$ GeV–TeV, i.e. inflation at low energy scales.

We have discussed a post-inflationary reheating scenario based on holographic thermalisation. The reheating temperature is given by (16), which is to be compared with the perturbative decay scenario (1) or the instant reheating scenario (17). The strongly coupled CFT that models the particle theory of the Universe is characterised by the AdS radius L, which encodes the gauge coupling and the number of colours N. This scenario is expected to be useful when strongly coupled dynamics dominates. There are many issues to be investigated further; to conclude, we comment on some of them. While our model is based on the spherical shell collapse which is consistent with the closed FRW universe, the expression for the reheating temperature (16) is independent of the curvature radius and thus applies to the flat universe as well. In particular, computation in the Poincaré coordinates is also possible, with the tension of the black brane given by the energy density of the inflaton. Extension of the holographic model to the open FRW, however, may encounter difficulties related to topological issues [15]. It would also be interesting to see if our scenario can accommodate lepto/baryogenesis, presumably by including vector degrees of freedom in the AdS. Another direction that deserves serious investigation is string theoretical construction possibly with low energy confinement. For this, dynamical blackhole formation e.g. in the Sakai–Sugimoto model [16] may be promising. With a concrete D-brane model one may be able to discuss possible (de)confinement transition during (or after) reheating. Finally, quantitative study of our scenario requires numerical treatment similar to [17] but with an oscillating boundary condition for the collapsing field. Recent numerical studies of blackhole formation have uncovered rich structures, such as the turbulent instabilities of the AdS space. It would be interesting to see if these discoveries have cosmological relevance in the physics of reheating.

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